Teaching First-degree Equations: Opportunities for developing Computational Thinking concepts

Luciana Leal da Silva Barbosa

Departamento de Informática
Instituto Federal de São Paulo, Birigui, São Paulo, Brasil
luciana.leal@ifsp.edu.br

Marcus Vinicius Maltempi
Departamento de Estatística, Matemática Aplicada e Computação
UNESP - Universidade Estadual Paulista, Rio Claro, SP, Brasil
marcus.maltempi@unesp.br

ABSTRACT
This work aims to present a hands-on activity for the first-degree equations. The activity was applied to a reinforcement class of the K-8, in a public school in Birigui, São Paulo, Brazil. The activity used the Balance of Equations game, built on the Scratch programming environment with the objective of teaching the initial ideas of first-degree equations to a class with a learning gap in this subject. Seven concepts related to computational thinking with strong potential to be worked during the activity were identified and analyzed how they could be explored through hands-on activity for the teaching of equations. The analysis showed that hands-on activity proposed has great potential to explore the seven skills identified here, thus contributing to the Computational Thinking articulated to the teaching of mathematics to create a productive and creative environment for teaching and learning.

Keywords

Computational thinking; Balances of Equations Game; Mathematics Education; Hands-on Activity

1. DESCRIPTION

Computational Thinking (CT) has been featured in several publications of researchers in education and computer science, who advocate the inclusion of a curriculum focused on the study of computer science in basic education. Although there is no consensus on a single definition for computational thinking, its advocates agree that basic education can and should enjoy a number of benefits when articulated to it. Thus, from a study of the literature on the different conceptions about CT, we sought to identify in the proposed activity a set of concepts related to the CT with strong potential to be worked and developed with students. They are: 1) problems formulation, 2) abstraction, 3) simulation, 4) generalization, 5) automation, 6) low floor, high ceiling, and 7) dynamic modeling. Given this context, this paper presents the results obtained from the application and analysis of a hands-on activity, seeking to answer the following question: How can computational thinking be explored through a hands-on activity for the teaching of first-degree equations ($x + a = b$)?

In order to analyze the potential of concepts related to CT for the teaching of Mathematics, we developed a hands-on activity to be applied to K-8 class for the study of the initial concepts of first-degree equations with one unknown. This group is studying the system of linear equations during the regular Math classes. However, some students have a wide learning gap and are unable to follow these lessons. They present difficulties even in basic concepts of Mathematics, as in the four arithmetic operations. Faced with this, the school principal decided to adopt the following strategy: offer reinforcement classes at the same time as regular Math classes.

Reinforcement classes are taught by a computer teacher, who carries out a teaching internship as a prerequisite for graduating in a degree in Computer Science. Meetings can happen in any schoolroom that is available, meaning there is no allocated place for this purpose. For the preparation and administration of the classes, the teacher uses the books and manuals made available by the São Paulo State Board of Education.

Thus, our objective here is not to present results obtained from an empirical research, but rather to take advantage of a case study that portrays much of the reality of Brazilian public schools to explore, reflect and discuss ideas of teachers and researchers, who seek solutions for the chronic problem in Mathematics education in our country. In order to do so, we orient our approach from the perspective of seven pedagogical aspects related to the CT: 1) formulation of problems, 2) abstraction, 3) simulation, 4) generalization, 5) automation, 6) low floor and high ceiling, 7) dynamic modeling. This approach was the result of research on the literature and our experience in teaching programming to young children. The definition of this set of concepts for the CT is not intended to restrict discussions about a more objective and precise definition for the term, but on the contrary, it intends to open up more space for a process of discussion, reflection and criticism, giving greater emphasis on the contributions of the CT to creative teaching and learning of mathematics.

2. THE HANDS-ON ACTIVITY

The proposed activity for teaching the initial concepts of first-degree equations included the construction of the Balance of Equations game. The game aims to set an analogy between the operation of a balance scale and the concept of equivalence of a first-degree equation with an unknown. In this way we intended to apply the principle of the plate weighing to introduce the concept of equivalence through the game.

During the teaching process of this topic, the teacher must take care not to transform the process of solving the mechanical and exhaustive equations, avoiding the use of terms like "go there with minus", or "is multiplying, go there dividing". This type of approach emphasizes the memorization of algorithms without the true understanding of the underlying concepts. In addition, it helps to further distance mathematics from students' daily lives

A balance scale aims to achieve balance between the two sides of the scale. This is achieved when the weights of the objects present on both sides are equal, or when the sum of the weights of all objects placed on both sides is equal. In this way, the balance between the plates, or balance of the scales represents the equality of an equation. Students, then, need to understand that to maintain the balance of the scales, every action performed on one plate needs to be repeated on the other plate; otherwise the scales will suffer an imbalance.

The game created follows this reasoning. It includes as actors a balance scale, apples, paper bags, and a basket. It is divided into two phases: exploration and activities. During the exploration phase, the student aims to understand the operation of a scale of dishes and to define an analogy between the balance of dishes and equivalence in a first degree equation. The second phase offers students various activities to study and solve equations through the scale. In this case study, we used only the first phase of the game, and due to time constraints it was possible to apply only two of the three phases of the planning of the teaching.

The lesson plan consisted of three phases. The first one to introduce and formalize the principle of balance of the scales. The second, with the purpose of discovering how many apples existed in the interior of paper bags placed on one of the plates. The third uses the game as a dynamic modeling system by manipulating variables and code commands. The three phases were organized into activities and are described below.

The principle of equilibrium

1. Game exploration: The goal of this phase is to allow an exploratory contact with the game in order to reach an understanding of the principle of equilibrium in an intuitive way, using only logical reasoning. After guiding them as to the rules of the game, it is up to the teacher to ask the students to set and remove apples from the plates freely.

2. Reflection: Here the teacher can question students about what is happening when they add and remove apples from the plates. Students should note that the scales are imbalanced or remain in balance depending on the number of apples placed on each plate.

3. Formalization: After obtaining the students' answers, the teacher can complement the reasoning of the students in order to formalize the principle of balance.

At this moment, students were able to easily understand the principle of balance, just observing what was happening and laughing at their own conclusions. When the teacher systematized the concept, the students had already understood it.

How many apples are in the paper bag?

Once the balance principle has been understood, the teacher asks students to include a paper bag in the left-hand platform. It has an unknown number of apples inside, and the student's goal is to find out how many apples are in it. Students should then manipulate the apples of the basket, setting them and removing them from the plates until they can discover the enigma. The teacher should make it clear that the goal of the game is to leave the balance scale in a state of equilibrium, and thus discover the amount sought.

This exploration process allows students to set their own strategies in order to understand what is happening, answer questions, and find the solution to the puzzle. This creates a research environment with autonomy, through which the student seeks to solve the problem without worrying about algorithms or mathematical formalisms. In addition, the student's ability to create their own strategies with creativity is emphasized.

After this phase, students are expected to have defined their own strategy for solving the problems of finding the number of apples in each paper bag. Thus, the teacher can finish this moment by discussing the strategies found by the students, the results obtained for each problem, and finally, by systematizing a problem-solving algorithm in a group, so that the students understand that when applying the algorithm built to any problem of this nature, the solution found will be correct. After that, the algorithm becomes more meaningful for students.

Programming equations

With an algorithm built for solving problems, the next step consists of presenting parts of the game code to the students, proposing activities of manipulation and exploration of the program variables. At this stage, it is intended to give students the opportunity to formulate their own problems. For this, the teacher should guide them to change the value of the variable that defines the number of apples inside each paper bag, and as a consequence, the amount of apples that can be placed in both plates. The student should understand that the quantities of apples placed on the plates depend on the value attributed to the variable paper bag. Otherwise, the balance will not be reached.

Each student's hypothesis can be built and tested instantly, by simply changing the variables and executing the code. This process of thinking and thinking about thinking, based on results obtained, is only allowed thanks to the environment built on the Scratch programming language, which gives the opportunity not only to program, but to change its code and to view immediately the results of these changes. In this way, a dynamic modeling system is created, used to formulate hypotheses, to test them, to analyze the results of the tests, thus validating or rectifying the original idea.

The third phase of the activity can be closed by systematizing the process of problem formulation. The teacher should organize and socialize the ideas of suggested problems, discuss the strategies used to formulate each one, evaluate their correctness, and finally, present to the students the systematization of problems through mathematical language, thus introducing algebraic language. Unfortunately, this last stage of planning could not be applied due to lack of time.

At the end of the three phases explained above, the teacher can begin the algebraic explanation of equations. The student should be able to recognize in the algebraic language and in the process of solving the equations the same strategies and algorithms defined and used in the previous steps.

3 A CLASSROOM CASE

The activity could not be fully applied to the eighth grade reinforcement class due to time constraints. The team of researchers and teachers had available only two meetings, a total of 4 lessons of 50 minutes each. In this section we will discuss these two meetings.

In addition to the first author of this text, a math teacher from the partner school and the trainee teacher responsible for the reinforcement classes participated in the meetings. During the two classes the following activities were developed:

1. Diagnostic evaluation with students.

Given a equation x + a = b, we asked the students about: what is an equation for? What does the letter "x" mean in the equation? What does "solving this equation" mean? How to solve this equation? The intention was to verify if an equation had any meaning for the students, and not only if they would be able to apply a solving algorithm to find the "x".

To our surprise, none of the students ventured an answer. In fact, they said they did not know the answers. Faced with this, we began the next step.

2. Introduction of the theme Balance of equations.

We have begun this step with some jokes to "break the ice" and introduce the theme "balance scale". We called some students in front of the class to illustrate the difference between the children's weights and asked what would happen if people with different weights climbed on opposite sides of a seesaw. Intuitively and based on their own experience, they were able to answer the questions correctly. We then set out for the first phase of the game, exploration.

3. Exploration of the Balance of Equations Game

This step began with the explanation of the rules of the game: "right arrow" key to set apples on the right weighing platform; “the left arrow” key to set apples on the left platform; "Up arrow" removes apples from the right platform; "Down arrow" removes apples from the left. We asked the students to set and freely remove apples from the plates. Each plate has an apple counter positioned just below. Then, as they put or took the apples, the counter increased or decreased its value, thus quantifying the apples on the platforms. In addition, the balance scale changed state depending on this quantity, being able to assume one of three states: balance, right imbalance, left imbalance.

After a few minutes of free exploration, we asked the students about their perceptions, what was happening when they set and took the apples off the plates. Their answers correctly described the facts, and in order to formalize the principle of equilibrium, we asked about the conditions necessary for the plates to remain at the same level. Again, they answered correctly without hesitation. In this way, phase 1 of the planning was completed successfully: the students were able to understand the operation of a scale and the necessary conditions to keep it in balance. Then we moved on to phase 2: how many apples are inside the paper bags?

4. How many apples are in the paper bags?

Then we asked them to put the first paper bag on the left weighing platform. Before that, we had explained that in each bag there was an unknown quantity of apples, and that now our goal was to find out how many apples existed inside. Introducing a puzzle during class is something that instigates children's curiosity, motivating them to search for answers in order to achieve the pleasure of discovery.

By setting the paper bag on the balance scale, it changed its position to left imbalance. Thus, the students realized that it had some "weight", and that they would then need to find out how many apples existed there. We suggested that they manipulate the apples that were out of the bag and on the two plates. We gave the following tip: leave the paper bag alone on the left plate, and we reminded them that the goal of the game was to keep the balance scale in equilibrium. Thus, the result was found, that is, at the end of the process they managed to keep the balance scale in equilibrium and discover the value sought.

Upon hearing their answers about this value, we asked them how they came to this conclusion. Again, they explained the path and justified their answers. This shows us that, no matter how much difficulty children have in doing traditional mathematics, they are able to develop successful intuitive and experimental reasoning. Moreover, the reasoning developed by the children made use of the principle of equilibrium explored and formalized previously, without its being introduced in an algorithm memorized and applied without signification.

5. Exploring the code: programming the values of x

In a third phase, we began the process of exploring the code in order to program different "weights" for the paper bags. This phase could not be finished due to lack of time. The central idea is to transform the Balance of Equations game into a dynamic modeling system, that is, to present students with the possibility to formulate their own problems (equations), test, visualize test results, analyze, reflect, and correct if necessary.

The Figure 1 illustrates one of the code excerpts from the Balance of Equations game presented to students. The proposed activities suggest students the change of the variable *saco1*, which represents the number of apples inside the character Paper Bag 1. It represents the value of the variable x that should be calculated in a first-degree equation. Such knowledge had already been formalized by the teacher in step 2 of the game. In this way, students should choose a value for this variable and, from this value, set the corresponding amounts of apples for the left and right plates.

For example, if the value chosen for saco1 is 12, how many apples can be added to the right and left plates so that the balance is steady? In this example the student may discover a combination of possibilities. For example, 5 and 17, 6 and 18, 7 and 19, for the left and right plates, respectively. He must realize that his choices must always respect the principle of equilibrium, and, as a consequence, the value assigned to the variable saco1 added to the value set for the left plate should result in the apples value of the right one. Facing this fact, students can formulate different problems (equations), one for each combination of defined values. Figure 3 shows the snippets of the code that defines the values of the left and right plate variables that should be manipulated by the students. All the learning situations discussed so far involved first-degree equations in the form x + a = b. However, the game also deals with situations involving the removal of apples from the paper bag, that is, equations in the form x - a = b. In these cases, the student should realize that the presence of the subtraction operation will result in the application of this same operation on the right weighing plate of the scale. This understanding will also come as a result of the process of simulation and exploration of the game. The teacher can then formalize the concept of inverse operations from these examples. It is intended to add other learning situations to the game to include equations of type ax + b = c.

4 ANALYZES AND CONCLUSIONS

This work aimed to present a hands-on activity planned for the teaching of first-degree equations with an unknown, analyzing its relation with some concepts related to CT, under the perspective of possibilities for a practical, creative and meaningful mathematical doing. Our approach defined a set of seven possibilities for CT found in the literature, reflecting how each of these concepts could be developed during the proposed activity and what their impact on the teaching and learning process of Mathematics.

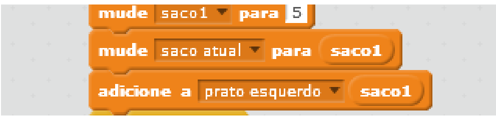

Figure 1: Excerpt of the code that defines the value of the variable *saco1*, which means defining the quantity of apples inside the paper bag 1.

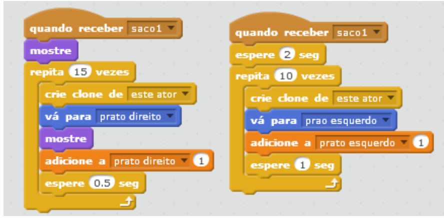

Figure 2: On the left, an excerpt of the code that defines the value of the right plate variable. In the example we have that right plate is equal to 15. On the right, excerpt of the code that defines the value of the left plate variable. In the example we have that left plate is equal to 10

The Balance of Equations game transforms into a dynamic modeling system when it gives students the opportunity to change variables or model parameters and visualize the results of these changes instantly. This happens during classes at a time when children are invited to change the values of the variables representing the paper bags, the right and the left weighing plates. Experimenting with code modifications provides practical experience with concepts, challenging the class to define and answer questions about roles and the impact of various parameters.

From the curriculum viewpoint, we realized that although the Balance of Equations game was developed for the purpose of teaching early concepts and ideas of first degree equations to an eighth grade reinforcement class, the high ceiling characteristic was identified in this resource, opening opportunities for teaching and learning of other concepts and mathematical relationships more complex and studied at more advanced levels. This feature gains even more force and importance when analyzed from the perspective of dynamic modeling, which enables low floor, high ceiling, abstraction, automation, simulation and generalization. In addition, the possibility of working with high ceilings opens space for teaching more complex ideas to young children, challenging the prevailing conception in Brazilian education that young children are not capable of developing abstract thoughts, limiting themselves only to the concrete world of ideas.

Our analysis of the abstraction capacity related to the proposed activity starts from the perspective of coding. It is a fact that in order to understand a real-world problem and represent its solution through a program, it is indispensable to use the ability of abstraction. Thus, presenting the problem of maintaining the equilibrium of a balance scale through a program and making the code available for study and manipulation, means exposing the students to an abstraction process that starts from the understanding of a concrete problem abstracted to a game.

The proposed activity automates the operation of a balance scale. That is, the game abstracts the operation of a two-plate scale and automates this operation. This would be the students' first contact with automation in this activity. In addition, students are encouraged to formulate their problems (equations), which will be modeled and executed within the context of the game. Here also occurs the automation of the solution of a problem formulated by the students.

Addressing the initial concepts of first-degree equations using the Balance of Equations game has made the teaching and learning process more meaningful for students by bringing the object to be learned from the reality lived by students. They were able to experience mathematics before formalizing it. They were able to find solutions to problems already known using their experience and logical reasoning in a natural way. The participation of the students during the development of the activity and their correct answers to the questions of the teacher showed that they were able to learn what was proposed. When the teacher reverses this process, presenting formal mathematics first, it distances these concepts from the students' reality, thus making learning difficult. This methodology became even more important for the success of learning concepts when we take into account the reality of the class, that is, students who had already gone through the teaching of this subject without being able to learn it. In addition, we can cite the relaxed atmosphere created from the use of the game, which consists of a playful resource that attracts the interest and motivation of the students. They learned while they were playing.

Another fact that deserves attention is the effective participation of the students during the activities and discussions. Those students said they did not like math. Their experiences in the regular classes of this discipline were frustrating, creating a demotivating and indiscipline classroom environment, a fact that did not happen during the application of the activity here reported. In addition, making math more meaningful to students allows them to see the content with other eyes, being able to recognize how it can be used to solve real world problems, giving it due importance.

In a way, all the possibilities related to CT identified in the proposal could be explored in an interrelated way, that is, the same activity could address more than one concept of CT at the same time. We believe that this potential is due to the nature of the activity that founded all the phases of the lesson plan, namely coding. Because it brings together all the concepts related to CT here discussed, we believe that programming has an inherent potential for the development of these and other concepts related to computational thinking. In a world where social and environmental challenges are increasing, and where technologies are essential for modeling and solving problems, it is even more important to develop the competencies studied here in our apprentices. Otherwise, how will you be able to act in the 21st century society consciously and competently?

5. BIOS

Luciana L. S. Barbosa (panel´s member). Teacher of Basic, Technical and Technological Education at the IFSP. Graduation and Master in Computer Science by UNESP. She works as a teacher in Technical and High courses in the area of computing, and in Colleges degree in Math and Physics. She works in projects with the objective of developing the computational thinking in children in K-12 through teaching of computer programming. She researches in the area of Informatics Education and Computational Thinking in math education.

Marcus V. Maltempi. Graduation in Computer Science, Master in Computer Science and Computational Math, PhD in Electrical and Computer Science, Professor´s Degree by UNESP. He works in the area of Informatics in Education, with emphasis in Math Education.